

1. What is meant by deduction? by induction? Give an example in which reasoning by induction leads to a false conclusion.
2. What is the difference between ordinary induction and "mathematical" or "complete" induction?
3. Prove the following by mathematical induction.
 - (a) $1 + 2 + 3 + 4 + \dots + n = \frac{n}{2}(n + 1)$.
 - (b) $2 + 4 + 6 + \dots + 2n = n(n + 1)$.
 - (c) $3 + 6 + 9 + \dots + 3n = \frac{3n}{2}(n + 1)$.
 - (d) $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$.
 - (e) $2^2 + 4^2 + 6^2 + \dots + (2n)^2 = \frac{2n(n+1)(2n+1)}{3}$.
 - (f) $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$.
 - (g) $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$.
 - (h) $2 + 2^2 + 2^3 + \dots + 2^n = 2(2^n - 1)$.
 - (i) $x^{2n} - y^{2n}$ is divisible by $x + y$ if n is any positive integer.
4. Prove the Binomial Theorem by Math. induction. For what value of n is it true?
5. What are the three laws of logic, according to Aristotle?
6. What ~~are~~ is "the starting point of any mathematical science?" Is there any choice about the starting point?
7. What is meant by postulate? by axiom? Illustrate.
8. What are the definitions, postulates, and axioms stated by Euclid? Criticise these, and indicate one postulate used by Euclid but not stated.
9. Discuss some of the attempts to prove Euclid's parallel postulate.
10. Show how we arrive at geometries other than Euclid's geometry.
11. Discuss the notion of a class, giving examples.
12. What is meant by the word "obvious?"
13. What is meant by "metric geometry?"
14. In Hilbert's assumptions for a geometry, what are the undefined terms? Discuss his sets of assumptions.
15. What is the "Archimedean assumption?" Give an example of a non-Archimedean system. (Young, Fundamental Concepts, p.149 et seq.)
16. What are the undefined terms in Veblen's assumption for geometry? in Pieri's?
17. What are the "two fundamental properties" which any set of assumptions for a mathematical science should possess?
18. What are the advantages, if any, in an abstract formulation of a branch of mathematics?

Ch.VII. FUNDAMENTAL IDEAS IN EUCLIDEAN
AND NON EUCLIDEAN GEOMETRIES

1. Preliminary notions.

- a. Inductive reasoning and "mathematical induction."
- b. Deductive reasoning.
- c. What is a "postulate," a "definition," when a "proof,"
Are axioms "self-evident" or "necessarily true?"
- d. Laws of logic.
 - i. Aristotle and the historic background.
 - ii. The three laws of logic.
 - a) Identity
 - b) Excluded middle
 - c) Contradiction
 - iii. Recent excursions into variations in ways of thinking.
- e. Deductive Science consists of
 - i. Set of undefined terms
 - ii. Set of unproved terms- propositions
 - iii. Theorems deduced from these using certain previously agreed upon laws of logic.
- f. Set of assumptions should have following properties.
 - i. Consistency
 - ii. Independence
 - iii. Sufficiency
- g. "Arbitrary assumptions" and "sensible statements."

2. Euclidean geometry

- a. Historic background
- b. Euclid's undefined terms, postulates, and axioms.
 - i. Criticism of Euclid's treatment.
 - ii. Some assumptions not mentioned by Euclid.
- c. Some paradoxes due to omissions in postulates.

3. Euclid's Parallel Postulate

- a. Methods of attack upon it.
 - i. Attempts to avoid difficulty through new definition of parallel line.
 - ii. New assumptions considered less faulty.
 - iii. Attempts to deduce theory of parallels from Euclid's other postulates by reasoning upon straight line and the plane angle.
 - iv. Maintain other postulates, deny this one, then if this one dependent upon others, a contradiction will be arrived at.
- b. Historic sign posts on the road to non*Euclidean geometry
 - i. Work of John Wallis.
 - ii. " " Saccheri.
 - iii. " " Gauss.

4. Non-Euclidean geometries.
 - a. Lobatchewsky and Bolyai assumptions.
 - b. Riemann assumptions.

5. Other sets of assumptions for Euclidean geometry.
 - a. Hilbert's assumptions *- German*
 - i. Undefined terms (five)
point, line, plane, between, congruent.
 - ii. Unproved propositions (twenty one)
 - a. Assumptions of alignment
 - b. " " order (Euclid omitted these) *Parish*
 - c. " " congruence
 - d. Axiom of parallels
 - e. Assumption of continuity.
Archimedean axiom and non-Archimedean system.

 - b. Veblen's assumptions *- American - Princeton University P.E.D.*
 - i. Undefined terms (two)
point, between.
 - ii. Unproved propositions (twelve)
 - a. Assumptions of order (six)
 - b. " " congruence (five)
 - c. " " parallels (one)

 - c. Pieri's assumptions
 - i. Undefined terms (two)
point, rigid motion (class of motions forms a group.) Definition of straight line and plane.
 - ii. Unproved propositions (thirteen).

worked with Moore.

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